IV. Normal Tables.

There are two separate classes of problems which you might encounter when using normal tables:

- Finding the percentile corresponding to a given score.
- Finding The score score corresponding to a given percentile.

These problems are important not because percentiles are intrinsically interesting – indeed as we have already seen, percentiles are statistical dead ends. The problems are important because they relate directly to skills needed to work other problems in inferential statistics.

We will deal will each problem as a separate scenario.

Scenario I (finding the percentile corresponding to a given score). You will be given a score and, based on data on means and standard deviations, will seek the percentile which corresponds to that score.

Data and parameters. You will be given either census or sample data regarding the mean and standard deviation. You will also be given a particular score – or observation – from the population.

Mean	$\boldsymbol{\mu}$ or $\bar{\boldsymbol{x}}$
StDev	$\boldsymbol{\sigma}$ or $\boldsymbol{s}$
Observation	$\boldsymbol{x}$

Research Objective. To find the percentile which corresponds to the given score.

Solution Template

Step 1. Make a list of what you are given. In some problems you will be given *census* data (as in the ACT problem above); in other problems you will only be given *sample* data, in which case you will use the sample data to *estimate* the population paramenters $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$.

mean	$\boldsymbol{\mu}$ or $\bar{\boldsymbol{x}}$
StDev	$\boldsymbol{\sigma}$ or $\boldsymbol{s}$
Observation or score	$\boldsymbol{x}$

Step 2. Use the formula

$$\boldsymbol{z} = \frac{\boldsymbol{x} - \boldsymbol{mu}}{\boldsymbol{\sigma}}$$

to convert the observation $\boldsymbol{x}$ to a $\boldsymbol{z}$ score. If you are only given sample data, you will need to approximate the formula with:

$$\begin{aligned} \boldsymbol{z} &= \frac{\boldsymbol{x} - \boldsymbol{\mu}}{\boldsymbol{\sigma}} \\ &\approx \frac{\boldsymbol{x} - \bar{\boldsymbol{x}}}{\boldsymbol{s}} \end{aligned}$$

Step 3. Find the proportion corresponding the $\boldsymbol{z}$ score in step 2 in the normal table starting. You do this by locating the $\boldsymbol{z}$ score starting in the left hand column ("outside-in").

Step 4. (Optional) Convert the proportion you find in step 3 to a percentile.

End of Solution Template

Example. Suppose that a normal population has a mean of 5 and a standard deviation of 4. Find the percentile corresponding to a score of 11.2.

Solution.

Step 1. You have been given the following data:

Mean	$\mu = 5$
StDev	$\sigma = 4$
Observation	$x = 11.2$

Step 2. Now use the formula to find the z-score:

$$\begin{aligned} z &= \frac{x - \mu}{\sigma} \\ &= \frac{11.2 - 5}{4} \\ &= 1.55 \end{aligned}$$

Step 3. Now in the normal tables look down the left-most column until you find 1.5 (actually, $1.5z$ – you will fill in the last place, the "z", by looking across this row). Now move across this row until you come tothe column headed by the number "5;" the entry in the table is 0.9394.

z	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1.5z	**0.9332**	**0.9345**	**0.9358**	**0.9370**	**0.9382**	**0.9394**	**0.9406**	**0.9418**	**0.9430**	**0.9441**

Step 4. Now converting this decimal to a percentile gives 93.94%. ▮

Example. Suppose that a normal population has a mean of 100 and a standard deviation of 10. In a sample of 500 scores, how many would you expect to fall below 118.6?

Solution. This is really a percentile problem: if you know the percentage of scores falling below 118.6 you can multiply this percentage times the total number of scores (500) to get the number you would expect to fall below 118.6.

Step 1. In this problem

Mean	$\mu = 100$
StDev	$\sigma = 10$
Observation	$x = 118.6$

Step 2. Using the formula gives a z score of

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &= \frac{118.6-100}{10} \\ &= 1.86 \end{aligned}$$

Step 3. Now in the normal tables look down the left-most column until you find "1.8z." Now move across this row until you come to the column headed by the number "6;" the entry in the table is 0.9686.

z	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1.8z	**0.9641**	**0.9649**	**0.9656**	**0.9664**	**0.9671**	**0.9679**	**0.9686**	**0.9693**	**0.9700**	**0.9706**

Step 4. Converting this decimal to a percentile gives 96.86%.

To complete the problem, you would expect that 96.87% of the scores would fall below 118.6. Since 96.86% of 500 is

$$0.9686 \times 500 = 484.3$$

you would expect approximately 484 of 500 scores to fall below 118.6 ▮

Example. Suppose that a normal population has a mean of 500 and a standard deviation of 100. Find the percentile corresponding to a score of 438.

Solution.

Step 1. In this problem

Mean	$\mu = 500$
StDev	$\sigma = 100$
Observation	$x = 438$

Step 2. Using the formula gives a z score of

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &= \frac{438-500}{100} \\ &= -0.62 \end{aligned}$$

Step 3. Since the z-score is negatve, you need to look in the negative part of the z-table. In the normal tables look down the left-most column until you find -0.6 (actually, $-0.6z$ – you will fill in the last place, the "z", by looking across this row). Now move across this row until you come to the column headed by the number "2;" the entry in the table is 0.2676.

z	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
-0.6z	**0.2742**	**0.2709**	**0.2676**	**0.2643**	**0.2611**	**0.2578**	**0.2546**	**0.2514**	**0.2482**	**0.2451**

Step 4. Converting this decimal to a percentile gives 26.76%. ∎

Scenario II (finding the score corresponding to a given percentile).

You will be given a percentile and, based on data on means and standard deviations, will find the score which corresponds to that percentile.

Data and parameters. You will be given either census or sample data regarding the mean and standard deviation. You will also be given a percentile

Mean	μ or $\bar{x}$
StDev	σ or s
Proportion	p

If you are given a percentile instead of a proportion, you must first convert it to a proportion.

Research Objective. To find the score which corresponds to the given proportion.

Solution Template

Step 1. Make a list of what is known. Once again, sometimes you will be given census data and sometimes you will be given sample data. In the latter case, you will use the sample estimates $\bar{x}$ and s to estimate μ and σ.

Mean	μ or $\bar{x}$
StDev	σ or s
Proportion	p

Don't forget to convert the given *percentile* to a *proportion* by dividing by 100%.

Step 2. Locate – as closely as possible – the given proportion inside the normal tables. If the proportion is greater than 0.5, look in the "positive" part of the table. If the proportion is less than 0.5, look in the "negative" part of the table. Reading "inside-out" find the corresponding z score.

Step 3. Compute the corresponding "raw" score by using the formula

$$x = \mu + z \times \sigma.$$

If μ and σ are not known, you must approximate μ and σ with the sample mean and standard deviation:

$$\begin{aligned} x &= \mu + z \times \sigma \\ &\approx \bar{x} + z \times s \end{aligned}$$

to obtain an approximate conversion to a raw score.

End of Solution Template

Example. Suppose that a normal population has a mean of 200 and a standard deviation of 30. What score corresponds to the 95th percentile.

Solution. In this problem Step 1. In this problem

Mean	$\mu = 200$
StDev	$\sigma = 30$
Proportion	$p = 0.95$

Note that we have converted the percentile 95% to a proportion 0.95.

Step 2. We would like to find 0.95 inside the table, but this exact number does not appear; thus we will use the number which does appear and is closest to 0.95. *Inside* the normal table (*not* in the left column) find the entry nearest to 0.95; this is either 0.9495 or 0.9505. Either value is acceptable, since both are equidistant from 0.95; we will use the values corresponding to 0.9495.

z	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1.6z	**0.9452**	**0.9463**	**0.9474**	**0.9485**	**0.9495**	**0.9505**	**0.9516**	**0.9526**	**0.9535**	**0.9545**

The number 0.9495 is in the row corresponding to "$1.6z$" and is in the column corresponding to "4." Thus the proportion 0.9495 corresponds to a z score of 1.64.*

Step 3. Now you are ready to use the formula

$$\begin{aligned} x &= \mu + z \times \sigma \\ &= 200 + 1.64 \times 30 \\ &= 200 + 49.2 \\ &= 249.2 \end{aligned}$$

Example. Suppose that a normal population has a mean of 50 and a standard deviation of 25. What score corresponds to the 25th percentile?

Solution.

Mean	$\mu = 50$
StDev	$\sigma = 25$
Proportion	$p = 0.25$

Converting 25% to a proportion gives $p = 0.25$.

Step 2. *Inside* the normal table (*not* in the left column) find the entry nearest to 0.25; this is either 0.2514 or 0.2482. In this case, the nearer of the two is 0.2514, so this is the value we will use.

z	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
-0.6z	**0.2742**	**0.2709**	**0.2676**	**0.2643**	**0.2611**	**0.2578**	**0.2546**	**0.2514**	**0.2482**	**0.2451**

The number 0.2514 is in the row corresponding to "$-0.6z$" and is in the column corresponding to "7." Thus the proportion 0.2514 corresponds to a Z score of -0.67.

* Of course, since 0.9505 corresponds to a z score of 1.65, we could use 1.645 as our Z value. This process, called "interpolation," is discussed in the text, but will not be covered in this course.

$\boxed{\textit{Step 3.}}$ Now you are ready to use the formula

$$
\begin{aligned}
\boldsymbol{x} &= \boldsymbol{\mu} + \boldsymbol{z} \times \boldsymbol{\sigma} \\
&= 50 + (-0.67) \times 25 \\
&= 50 - 16.75 \\
&= 33.25
\end{aligned}
$$

Problems

1. In a normal population having a mean of 20 and a standard deviation of 5, esitmate the percentile corresponding to an observation of 22.

2. In a normal population having a mean of 20 and a standard deviation of 5, esitmate the percentile corresponding to an observation of 28.

3. In a normal population having a mean of 500 and a standard deviation of 100, estimate the score corresponding to a percentile of 75.

4. In a normal population having a mean of 550 and a standard deviation of 16, esitmate the percentile corresponding to an observation of 512.

5. In a normal population having a mean of 100 and a standard deviation of 15, estimate the score corresponding to a percentile of 25.

6. In a normal population having a mean of 20 and a standard deviation of 5, estimate the score corresponding to a percentile of $33\frac{1}{3}$.

7. In a normal population having a mean of 100 and a standard deviation of 15, esitmate the percentile corresponding to an observation of 148.

8. In a normal population having a mean of 100 and a standard deviation of 15, esitmate the percentile corresponding to an observation of 80.

Video Assignment.

View the following program(s) from the series *Against All Odds*:

Program	*Title*
Four	*Normal Distributions*
Five	*Normal Calculations*