XI. Analysis of Variance (ANOVA)

Scenario. You will have sample data from several populations. The observations will have been taken on each experimental group only once, after exposure to the experimental treatment.

√ *Given.* For purposes of illustration, we will suppose that you have data from three different treatments (A, B and C). The table would be similar if four or more treatments were applied.

	Sample A	Sample B	Sample C
means	$\bar{x}_A$	$\bar{x}_B$	$\bar{x}_C$
sample size	n_A	n_B	n_C

In addition you will be given information about the entire sample (pooling all treatment groups together).

overall sample mean x

pooled sample standard deviation s

√ *Research Objective:* To determine if there is any difference between the treatment groups in the response measured by the means. When doing the analysis of variance, the null and alternative hypotheses are always

$$H_O : \mu_A = \mu_B = \mu_C$$

H_A :at least two means are different

Solution Template

Step 1. First find the total variance of the pooled sample:

$$V_T = s^2$$

by squaring the pooled sample standard deviation.

Step 2. Next find the variance due to the column effects V_C.

- If the sample sizes are all equal, you can do this directly on your calculator:
  1. Place the calculator in `STAT` mode;
  2. Enter the sample means in your calculator as data (using the $\boxed{\Sigma+}$ key).
  3. Find the population standard deviation of the sample means (strike the $\boxed{\texttt{SECOND}}$ $\boxed{\sigma_n \; 6}$ keys).
  4. Square the result (strike the $\boxed{x^2}$ key). The displayed number is V_C.
- If the sample sizes are *not* all the same, use the formula:

$$V_C = \frac{n_A \bar{x}_A^2 + n_B \bar{x}_B^2 + n_C \bar{x}_C^2}{n_A + n_B + n_C} - \bar{x}^2.$$

Step 3. Construct the following table:

		degrees of freedom
Total Variation – Column Variation	V_T V_C	$n_A + n_B + n_C - 1$ # of columns - 1
Residual Variation	V_R	

The last row is the difference of the first two rows.

Step 4. Find the value of the test statistic:

$$F = \frac{(V_C/\text{deg. free. for } V_C)}{(V_R/\text{deg. free. for } V_R)}.$$

Step 5. Look up the appropriate cutoff in one of the Appendices F to this study guide. Use:

$$\begin{aligned} \text{Column} &= \text{degrees of freedom for the columns} \\ \text{Row} &= \text{degrees of freedom for } \boldsymbol{V_R} \end{aligned}$$

Notice that Appendix F is really several tables, depending on the significance level.

Step 6. Reject $\boldsymbol{H}_0$ if the value in step four is *larger than* the cutoff.

End of Solution Template

Interpretation. The results of this test will tell you whether or not at least one of the means is different from the others; it will not tell you *which one* is different. You would need to test each pair of means to find out if any two differ one from the other.

Assumptions. We must assume that each treatment population has a normally distribution.

Example. During the cooking process, doughnuts absorb fat in various amounts. In this problem, we will try to determine if the amount of fat absorbed depends upon the type of fat used. For each of four types of fats, six batches of doughnuts were prepared. The data in the following table summarize the average grams of fat absorbed per batch.

Fat type	A	B	C	D
means	172	185	176	162
# of batches	6	6	6	6

The overall mean was 173.75 grams absorbed per batch and the overall standard deviation was $\boldsymbol{s} = 12.34$.

Using a significance level of $\boldsymbol{\alpha} = 0.05$, do these data show that there is a difference in the amount of fat absorbed for the various types of fats?

Solution.

Step 1. The total sample variance $\boldsymbol{V_T} = 12.34^2 = 152.28$ grams.

Step 2. Since this problem has balanced samples between the four treatment groups, we could use our calculator to find the column variance $\boldsymbol{V_C}$. For illustrative purposes, we will use the formula instead (both give the same answer). Note that the formula is adjusted for four treatment groups.

$$\begin{aligned}
\boldsymbol{V_C} &= \frac{\boldsymbol{n_A \bar{x}_A^2 + n_B \bar{x}_B^2 + n_C \bar{x}_C^2 + n_D \bar{x}_D^2}}{\boldsymbol{n_A + n_B + n_C + n_D}} - \bar{\boldsymbol{x}}^2 \\
&= \frac{6 \times 172^2 + 6 \times 185^2 + 6 \times 176^2 + 6 \times 162^2}{24} - 30,189.06 \\
&= \frac{726,174}{24} - 30,189.06 \\
&= 68.19
\end{aligned}$$

Step 3. Construct the table:

		degrees of freedom
$\boldsymbol{V_T}$	152.28	23
$-\boldsymbol{V_C}$	68.19	3
$\boldsymbol{V_R}$	84.09	20

Step 4. Compute the test statistic:

$$\begin{aligned}
\boldsymbol{F} &= \frac{(\boldsymbol{V_C}/\text{deg. free. for } \boldsymbol{V_C})}{(\boldsymbol{V_R}/\text{deg. free. for } \boldsymbol{V_R})} \\
&= \frac{(68.19/3)}{(84.09/20)} \\
&= \frac{22.73}{4.20} \\
& 5.41
\end{aligned}$$

Step 5. The 5% cutoff in Appendix F (column 3, row 20) is 3.0984. (The 1% cutoff is 4.9382.)

Step 6. We can reject $\boldsymbol{H}_0$ since $5.41 > 3.10$. Thus, the amount of fat absorbed does vary with the fat used; which fat is absorbed more or less can be evaluated using a hypothesis test for the difference in means.

Problems

1. A researcher gathers the following data:

	A	B	C	D
means	16	18	23	17
count	50	50	50	50

If the pooled deviation of the sample is 13.8, find out if there is a statistically significant difference in the means for the above groups. Use a significance level of 5%.

2. A researcher gathers the following data:

	A	B	C	D	E
means	38	36	39	31	32
count	25	25	25	25	25

If the pooled deviation of the sample is 7, find out if there is a statistically significant difference in the means for the above groups. Use a significance level of 5%.

3. A researcher gathers the following data:

	A	B	C
means	523	478	521
count	22	31	19

If the pooled deviation of the sample is 76, find out if there is a statistically significant difference in the means for the above groups. Use a significance level of 5%.

4. A researcher conjectures that background music can influence the ability of people to concentrate. To test this conjecture, the researcher develops a short test of reasoning and mathematical skills and randomly selects 400 subjects. Each subject is then randomly assigned to one of four groups:

Group A: takes the test with no background music.

Group B: takes the test with a Bach cello suite playing in the background.

Group C: takes the test with show tunes from Rodgers and Hammerstein musicals playing in the background.

Group D: takes the test with the latest Pearl Jam album playing in the background.

The researcher gathers the following results:

	Group A	Group B	Group C	Group D
mean score	75	81	70	73
sample size	100	100	100	100

The overall standard deviation for all 400 scores was 26.

(i) Has the researcher gathered evidence to support the conjecture that background music effects the outcome of this examination? Use a significance level of 2%.

(ii) The pooled standard deviation for Groups A, C and D was 13.3. Is there any significant difference (at the 2% level) in the scores for these three groups?

(iii) What other factors which have been left uncontrolled in this experiment might have influenced the outcomes? How would you devise and experiment to control for these factors?

5. A group of researchers at Brigham Young University were interested in whether sex education programs which involve teaching abstinence (as opposed to teaching birth control) might effect attitude changes in high school juniors. To test this, they developed a survey to determine teen attitudes on sexual matters and also developed a sex education program which included teaching abstinence. They administered their attitude survey to three groups of high school juniors at randomly selected public high schools in the Salt Lake City area.

Group I: received no in-school sex education.

Group II: received conventional sex education without the special abstinence section.

Group III: received the conventional sex education with the specially designed section on abstitence.

The average scores on the researcher's attitude scale for each group are listed below (a higher score indicates that the student is more accepting of abstinence):

	Group I	Group II	Group III
means	110	114	118
sample size	50	50	50

In the overall group of 150, the standard deviation was 15.62

(i) Have the researchers found a significant difference in the attitude scores between the three groups? (Use a significance level of 5%.)

(ii) In an effort to explain their unexpected results, the researchers decided to disregard data from "deviant subject types" (such as those from broken homes or where the mother worked). Once the data were purged of these "deviants", they did obtain a significant difference in the three treatments. Comment on this research methodology. (This is a real study: I'm not making this up!)