7. Outside-In Calculations

7.1. Example.

Suppose that a population is normally distributed with mean 100 and standard deviation 15 (IQ scores are so distributed). Find the percentile which corresponds to an IQ score of 114.

Solution.
Since the normal curves are completely characterized by their mean and standard deviation, all we have to do is look in a table which gives the percentiles for scores taken from a population that is normally distributed having mean $\mu = 100$ and standard deviation $\sigma = 15$.
However, no such table exists. There is only *one* normal table: the one

for populations that have mean $\mu = 0$ and standard deviation $\sigma = 1$ – the "standard normal" or z table.
To solve this problem, we need to convert the "raw" score of 28 to a "standard normal" or "z" score; then we can use the normal table which appears in your study guide.
Step 1. First *make a dictionary:*

mean	μ	100
StDev	σ	15
Score	x	114

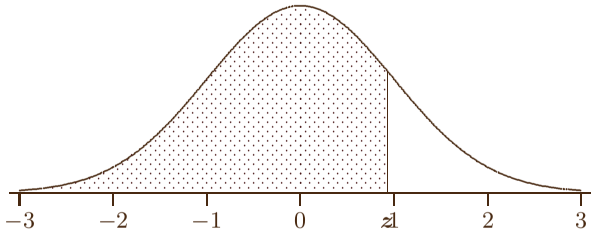
Step 2. Next convert the given score (or observation) to a *standard normal* or z score using the formula:

$$z = \frac{x - \mu}{\sigma}$$

In our example,

$$z = \frac{114 - 100}{15} = \frac{14}{15} = 0.93$$

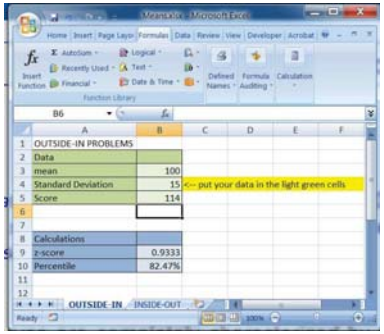
Step 3. Now look at the normal table in your study guide. There are actually two normal tables, one for negative values of z and one for positive values of z. Since our $z = 0.93$, we will look in the part of the table corresponding to positive values of z. The table gives the proportion of observations which fall to the left of z:



In this case, we need to look up the proportion corresponding to an observation of $z = 0.93$. To do this, look in the left column of the table until you find the first two digits; you will actually see $0.9z$. Now move to the column corresponding to the last digit; the number which you find in

this column is the *proportion* of observations which are less than z.
The proportion which you should find in the table is 0.8238.
Step 4. (Optional) You can convert the proportion to a percentile by multiplying by 100%. The corresponding percentile is 82.38%, i.e., 82.38% of all IQ scores are 114 or lower.
There is a spreadsheet solution to this as well.

To use the spreadsheet, open MEANS.XLSX, found in the resources section for this course on LEARN.OU.EDU. Note that you will need to select the tab at the bottom labeled OUTSIDE-IN. Enter the data.
From this, the corresponding percentile is **82.47%**.



Solution Template

Step 1. Make a list of what you are given. In some problems you will be given *census* data (as in the ACT problem above); in other problems you will only be given *sample* data, in which case you will use the

sample data to *estimate* the population parameters μ and σ.

mean	μ or $\bar{x}$
StDev	σ or s
Observation or score	x

Step 2. Use the formula

$$z = \frac{x - \mu}{\sigma}$$

to convert the observation x to a z score. If you are only given sample data, you will need to approximate the formula with:

$$z = \frac{x - \mu}{\sigma} \approx \frac{x - \bar{x}}{s}$$

Step 3. Find the proportion corresponding the z score in step 2 using the normal table. You do this by locating the z score starting in the left hand column ("outside-in").
Step 4. (Optional) Convert the proportion you find in step 3 to a percentile by multiplying by 100%.

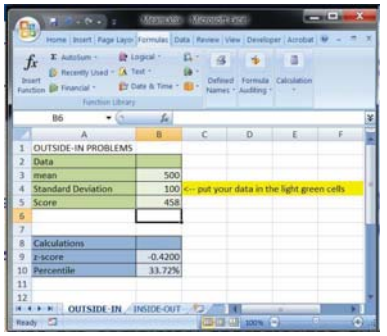
End of Solution Template

7.2. Example.

GRE scores are normally distributed with a mean of 500 and a standard deviation of 100. A student's GRE score is 458; what is the corresponding percentile?

Solution. Of course, the easy way to do this is with the spreadsheet.

To use the spreadsheet, open MEANS.XLSX, found in the resources section for this course on LEARN.OU.EDU. Note that you will need to select the tab at the bottom labeled OUTSIDE-IN. Enter the data.
From this, the corresponding percentile is **33.72%**.



For completeness, we'll include the methodology with the tables.
Step 1. In this problem

mean	$\mu = 500$
StDev	$\sigma = 100$
Observation	$x = 458$

Step 2. Find the z score;

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &= \frac{458-500}{100} \\ &= -\frac{42}{100} \\ &= -0.42 \end{aligned}$$

Step 3. Looking "outside-in" (but this time in the *negative* part of the z table) you can find that the corresponding proportion is 0.3372.
Step 4. In other words, 33.72% of all scores will be less than the observed score of 458.

Question. How many GRE scores would you expect to be *larger than* 458?

7.3. Example.

The Norman Speedskating Team has 18 members who skate at practices. For these 18 members, the average lap time is 12.1 seconds with a standard deviation of 1.38 seconds. Assuming that these data are from a normally distributed population, what percentile corresponds to a lap time of 11.8 seconds?

Solution. We'll do this one just with the spreadsheet.

To use the spreadsheet, open `MEANS.XLSX`, found in the resources section for this course on `LEARN.OU.EDU`. Note that you will need to select the tab at the bottom labeled `OUTSIDE-IN`. Enter the data.
From this, the corresponding percentile is **41.40%**.