## 8. Inside-Out Calculations

So far all we have discussed is the first of the two types of normal computations: you have been *given* a raw score (such as a lap time) and have been *seeking* the percentile which corresponds to that score. You have been working "outside-in" with the normal tables since you have been finding the $z$ score on the outside left edge of the table and then have been finding the corresponding proportion on the inside of the table.

Now we will consider the class of problem: you will be *given* a percentile and will be *seeking* the raw score which corresponds to that percentile. This class of problem will be "inside-out" since all of the steps are reversed.

## 8.1. Example.

*Recall that GRE scores are normally distributed with a mean of 500 and a standard deviation of 100. What GRE score corresponds to the 80th percentile?*

**Solution.** There is a spreadsheet solution to this as well, but we'll first do the longer way.

**Step 1.** Again, the first step is to make a list of what you know:

| | |
|---|---|
| mean | $\mu = 500$ |
| StDev | $\sigma = 100$ |
| Proportion | $0.80$ |

Note that since the table deals with *proportions* we have converted the percentile to a proportion by dividing by 100%.

**Step 2.** Now we need to find the $z$ score which corresponds to the given proportion of 0.80. To do this, we must first locate the proportion *inside* the body of the table.

Inside the table, you can't find 0.8000 exactly; you $can$ find

$$\begin{array}{ccc} 0.7996 & \text{and} & 0.8023 \\ \updownarrow & & \updownarrow \\ z = 0.84 & & z = 0.85 \end{array}$$

The $z$-score corresponding to 0.8000 is somewhere between $z = 0.84$ and $z = 0.85$. Since the table is only accurate to two decimal places, we can't do much better than this. We can see, though, that $z = 0.84$ corresponds more closely to 80% than does $z = 0.85$ (0.7996 vs. 0.8023). Hence we will use $z = 0.84$.

**Step 3.** The next step is to convert this $z$ score to a corresponding GRE score ("raw" score). To do this, you will need to use the formula

$$x = \mu + z \times \sigma$$

which is just the formula

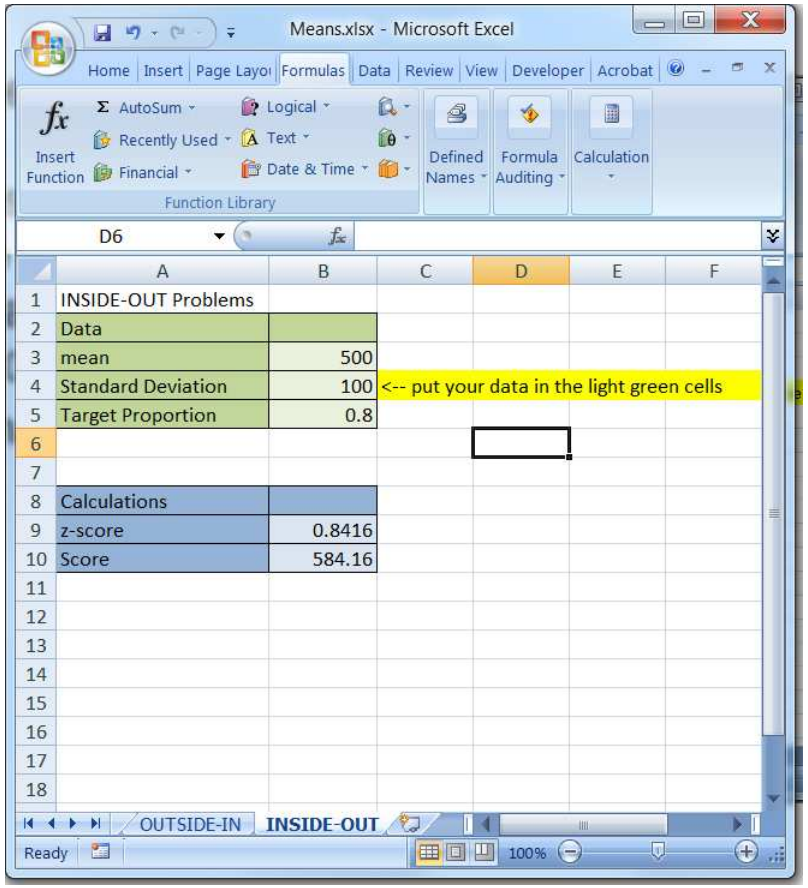
$$z = \frac{x - \mu}{\sigma}$$

solved algebraically for $x$. In our problem, this becomes

$$\begin{aligned} x &= \mu + z \times \sigma \\ &= 500 + 0.84 \times 100 \\ &= 500 + 84 \\ &= 584 \end{aligned}$$

and so a GRE score of 584 corresponds to a percentile of 80%.

To use the spreadsheet, open MEANS.XLSX, found in the resources section for this course on LEARN.OU.EDU. Note that you will need to select the tab at the bottom labeled INSIDE-OUT.
Fill in the data dictionary and read the result.



## Solution Template

**Step 1.** Make a list of what is known. Once again, sometimes you will be given census data and sometimes you will be given sample data. In the latter case, you will use the sample estimates $\bar{x}$ and $s$ to estimate $\mu$ and $\sigma$.

| | |
|---|---|
| Mean | $\mu$ or $\bar{x}$ |
| StDev | $\sigma$ or $s$ |
| Proportion | p |

Don't forget to convert the given *percentile* to a *proportion* by dividing by 100%.

**Step 2.** Locate – as closely as possible – the given proportion inside the normal tables. If the proportion is greater than 0.5, look in the "positive" part of the table. If the proportion is less than 0.5, look in the "negative" part of the table. Reading "inside-out" find the corresponding

$z$ score.

**Step 3.** Compute the corresponding "raw" score by using the formula

$$x = \mu + z \times \sigma.$$

If $\mu$ and $\sigma$ are not known, you must approximate $\mu$ and $\sigma$ with the sample mean and standard deviation:

$$\begin{aligned} x &= \mu + z \times \sigma \\ &\approx \bar{x} + z \times s \end{aligned}$$

to obtain an approximate conversion to a raw score.

**End of Solution Template**

**8.2. Example.**

*Find the ACT score which corresponds to the 25th percentile. (Recall that ACT scores have a mean of 20 and a standard deviation of 5.)*

**Solution.** **Step 1.** In this problem

| | |
|---|---|
| Mean | $\mu = 20$ |
| StDev | $\sigma = 5$ |
| Proportion | $p = 0.25$ |

**Step 2.** We must look *inside* the table for 0.2500. The "positive" part of the table gives $z$ scores greater than zero and hence proportions greater than one half; the "negative" part of the table gives $z$ scores less than zero and hence proportions less than one half. Thus, we can only find 0.2500 inside the negative part of the normal table.

We can't find 0.2500 exactly but we can find

$$\begin{array}{ccc} 0.2514 & \text{and} & 0.2482 \\ \updownarrow & & \updownarrow \\ z = -0.67 & & z = -0.68 \end{array}$$

Since the proportion corresponding to $z = -0.67$ is slightly closer, we will use $z = -0.67$.

**Step 3.** Now we can find the corresponding ACT score:

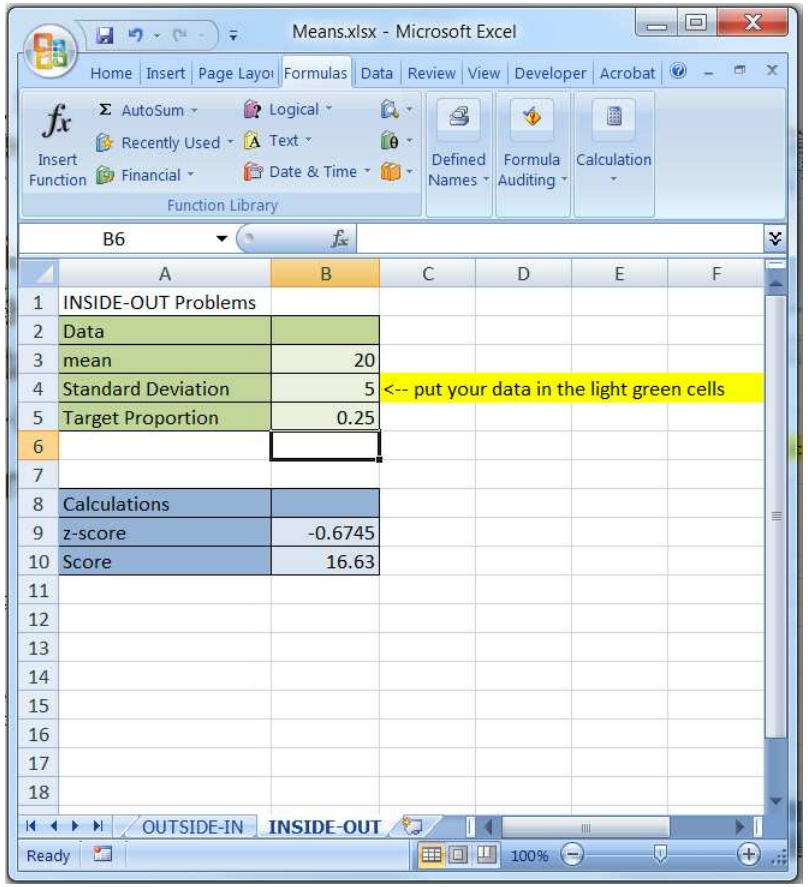
$$\begin{aligned} x &= \mu + z \times \sigma \\ &= 20 + (-0.67) \times 5 \\ &= 20 - 3.35 \\ &= 16.65 \end{aligned}$$

and so a score of 16.65 corresponds to the 25th percentile.

Alternatively, using the spreadsheet:

To use the spreadsheet, open MEANS.XLSX, found in the resources section for this course on LEARN.OU.EDU. Note that you will need to select the tab at the bottom labeled INSIDE-OUT.

Fill in the data dictionary and read the result.



## 8.3. Example.

*There are eight members of the Norman Speedskating team who have amateur cards and compete in regional meets; their lap times are*

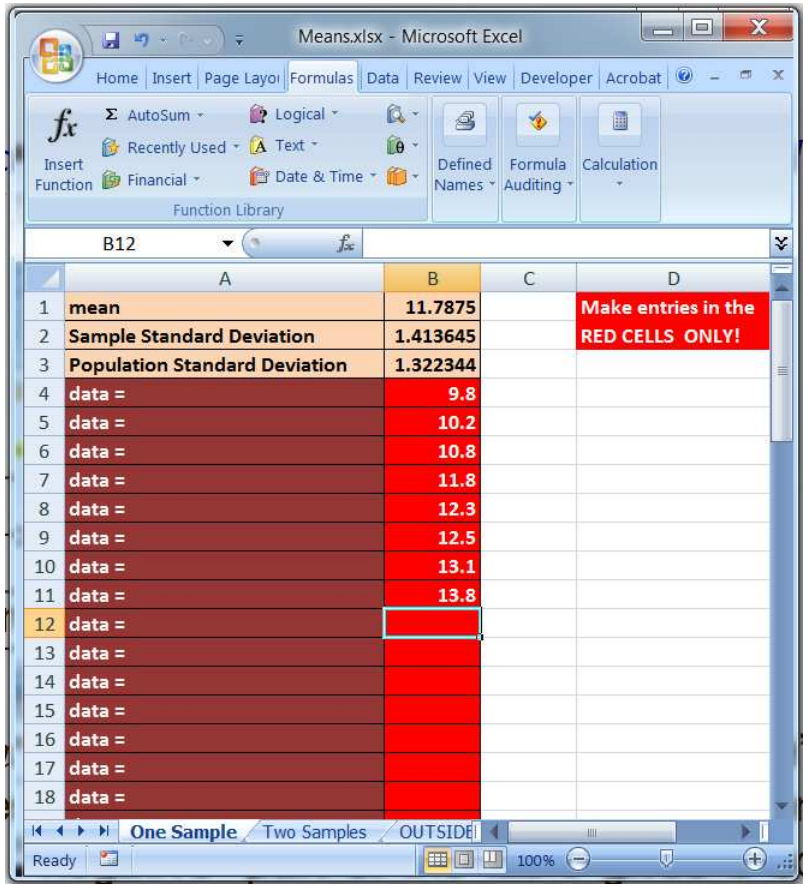
| | | | |
|---|---|---|---|
| *9.8* | *10.2* | *10.8* | *11.8* |
| *12.3* | *12.5* | *13.1* | *13.8* |

*(a) Find the mean and standard deviation of this sample.*

**Solution.**

*To use the spreadsheet, open* MEANS.XLSX, *found in the resources section for this course on* LEARN.OU.EDU. *Note that you will need to select the tab at the bottom labeled* INSIDE-OUT. *Enter the data.*

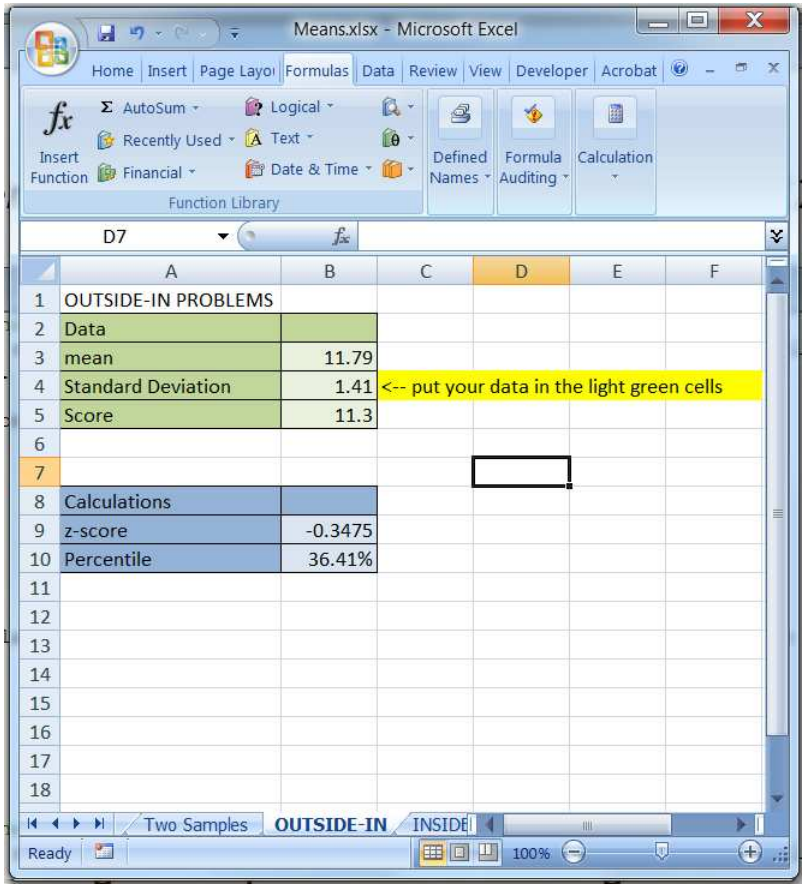
*From this, the mean is* $\bar{x} =$**11.79** *and the standard deviation is* $s =$**1.41**. *(What did you do wrong if you got 1.322 for the standard deviation?)*



*(b) Assuming that the data are from a normally distributed population, approximate the percentile which corresponds to a lap time of 11.3 seconds.*

**Solution.** This is an outside-in problem.

**Solution.**
To use the spreadsheet, open MEANS.XLSX, found in the resources section for this course on LEARN.OU.EDU. Note that you will need to select the tab at the bottom labeled INSIDE-OUT. Enter the data.
From this, the corresponding percentile is **36.41%**.



(*c*) Assuming that the data are from a normally distributed population, approximate the time which corresponds to the 99th percentile. This is an inside-out problem.

To use the spreadsheet, open `MEANS.XLSX`, found in the resources section for this course on `LEARN.OU.EDU`. Note that you will need to select the tab at the bottom labeled `INSIDE-OUT`. Enter the data.

From this, the corresponding lap time is **15.07 seconds**.