15. Confidence Intervals for Means

Idea: The sample mean $\bar{x}$ estimates the true population mean μ. A confidence interval assigns an "error bound" so that the true mean is probably between

$$(\bar{x} - \mathtt{ERROR}) \text{ and } (\bar{x} + \mathtt{ERROR})$$

Confidence intervals give a way of selecting the "error" so that a fixed percentage (say 95%) of all possible intervals

$$(\bar{x} - \mathtt{ERROR}) \text{ to } (\bar{x} + \mathtt{ERROR})$$

contain the true population mean μ.

The interval

$$\bar{x} \pm \mathtt{ERROR}$$

is called a *confidence interval*. Since this interval is based on sample data, we can't be *certain* that the population mean is inside this interval – we can only be "confident."

If you know the sample size, the sample mean, and the sample standard deviation, the Excel spreadsheet `Formulas.XLSX` in the resources section of `LEARN.OU.EDU` for this course will calculate the value of the error term for you according to the formula:

$$\mathtt{ERROR} = \pm \frac{zs}{\sqrt{n}}$$

where z is a cut-off found from the normal tables.

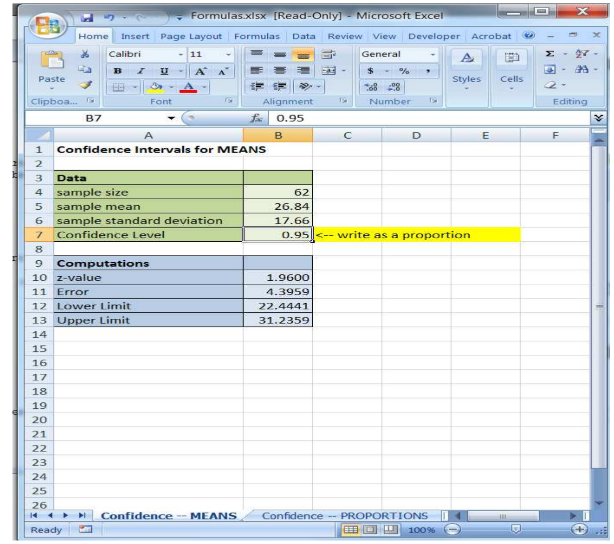
15.1. Example.

Suppose that 62 night shift workers are randomly selected and surveyed to find the weekly hours spent on child care. In this sample, it is found that the mean is 26.84 hours and the standard deviation is 17.66 hours. Construct a 95% confidence interval for the average weekly time spent in child care by night shift workers.

Solution. $\boxed{\textbf{Step 1.}}$ First make a list of all variables in the problem:

$\bar{x}$	26.84
s	17.66
n	62

Step 2. Now enter the list into the spreadsheet FORMULAS.XLSX, found in the resources section for this course on LEARN.OU.EDU. Note that you will need to select the tab at the bottom labeled `Confidence-MEANS`. Note also that you should convert the 95% to a proportion, 0.95.



Once you have entered the dictionary, the spreadsheet calculates the confidence interval for you:

$$22.41 \text{ to } 31.24.$$

We are "95%" confident that the true average hours spent on child care by night shift workers is somewhere between 22.41 hours and 31.24 hours.

∎

Questions. Suppose that you know that day shift workers spend, on average, 29.71 hours in child care. (Since we “know” this, 29.71 must be the true mean μ for day shift workers.) Is the above experiment evidence that night shift workers spend less time on child care than do day shift workers? Suppose that day shift workers spent 35 hours per week? What are some uncontrolled variables in this experiment?

Solution Template

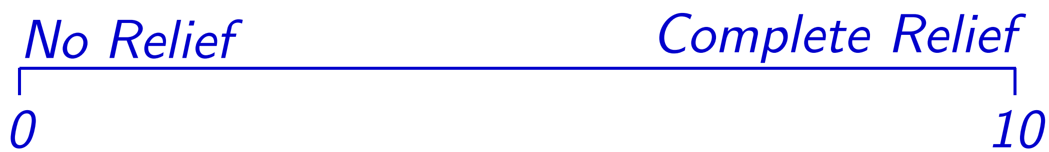
Step 1. Make a dictionary of the data given in the problem:

sample mean	$\bar{x}$
sample standard deviation	s
sample size	n
confidence level	C

Step 2. Use the Excel spreadsheet to calculate the confidence interval.

15.2. Example.

In a study on using hypnosis to relieve pain, 58 subjects were asked to fill out the following ***Likert Scale****:*

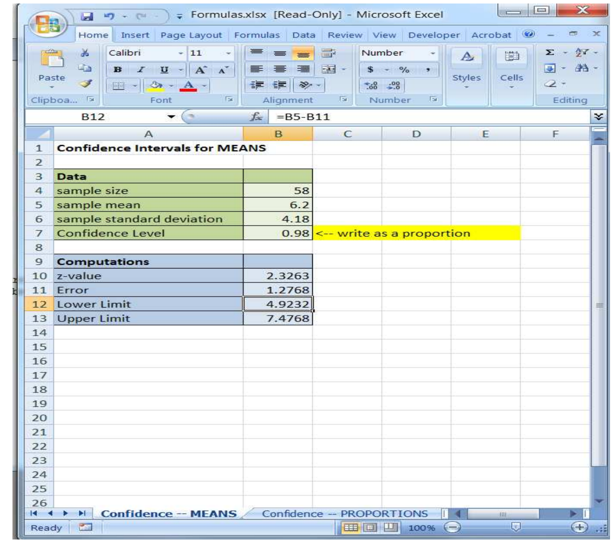


(The total length of the scale was 10cm.) For this sample, the mean was 6.2 and the standard deviation was 4.18. Find a 98% confidence interval for the perceived pain relief.

Solution. **Step 1.** First make a list of all variables in the problem:

$\bar{x}$	6.2
s	4.18
n	58

Step 2. Now enter the list into the spreadsheet FORMULAS.XLSX, found in the resources section for this course on LEARN.OU.EDU. Note that you will need to select the tab at the bottom labeled `Confidence-MEANS`. Note also that you should convert the 98% to a proportion, 0.98.



The resulting 98% confidence interval is

$$4.92 \text{ to } 7.48$$

Question. suppose we know that people with similar pain who take aspirin have mean pain relief of 7.42, as measured by this scale. Do we have convincing evidence that aspirin does better than hypnosis?

Suppose ibuprofen results in a score of 7.81; would we be able to conclude ibuprofen is better than hypnosis? Do we have convincing evidence that ibuprofen is better than aspirin? What kind of data would we have to gather to compare ibuprofen and aspirin?

16. Confidence Intervals for Proportions

Confidence intervals for proportions have the same basic underlying concepts as confidence intervals for means. The only difference is in the calculation that Excel uses for the error term:

$$\texttt{ERROR} = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where n is the sample size and $\hat{p}$ is the sample proportion:

$$\hat{p} = \frac{\text{number of successes}}{\text{sample size}}$$

Remember: "success" is the outcome which is the focus of the research and not necessarily any conventional notion of success or failure.

16.1. Example.

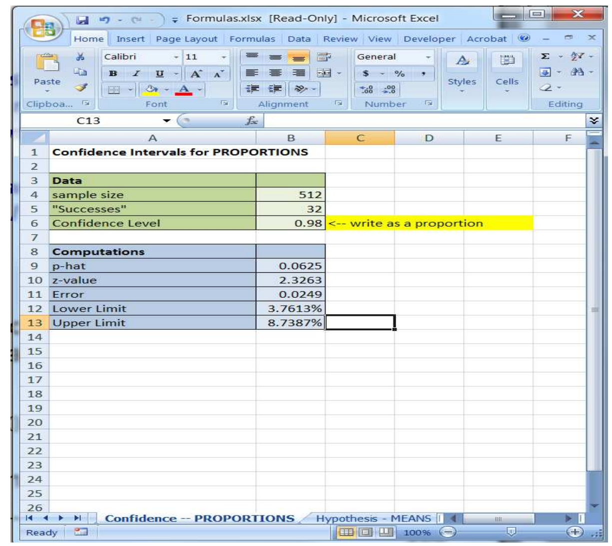
Currently all lab samples from a physician's office are sent to $\textit{Tests R Us}$, a commercial lab specializing in analyzing and producing pathology reports. The physician suspects that $\textit{Tests R Us}$ may be cutting corners, and decides to double check their results against the state laboratory which has essentially a 100% accuracy rate. Of 512 samples, a $\textit{Tests R Us}$ incorrectly identifies 32. Find a 98% confidence interval for the proportion of incorrectly identified samples.

Solution.

Step 1. First find the sample proportion; since problem is focusing on incorrect identifications, "success" is an incorrectly identified tissue sample.

sample size n	512
"successes" k	32
confidence level	98%

Step 2. Now enter the list into the spreadsheet FORMULAS.XLSX, found in the resources section for this course on LEARN.OU.EDU. Note that you will need to select the tab at the bottom labeled `Confidence-PROPORTIONS`. Note also that you should convert the 98% to a proportion, 0.98.



In particular, the confidence interval for the proportion is 3.75% to 8.47%.

Question. The physician could send all samples to different commercial lab, *Test Depot*. A recent analysis of their results indicated that *Test Depot* will incorrectly identify samples 4% of the time. However, *Test Depot* will charge nearly double per test compared with *Tests R Us*. Do we have sufficient evidence to justify spending more on *Test Depot*?

Remark. There are two kinds of errors which the lab could make:

- They could report no disease when the patient is really ill.
- They could report disease when the patient is healthy.

Question. Which kind of error do you think is more serious? Is it important to know which kind of error the lab is making?

$\longrightarrow$ In our next topic – hypothesis testing – there will always be two kinds of error which are possible. In general a researcher will be able to control the chances of only one of the two types of error. Part of the research design is to decide which kind of error is more important to control.

Solution Template

Step 1. Make a dictionary of the data given in the problem:

"successes"	k
sample size	n
confidence level	C

The formulae used by the spreadsheet require that both $n\hat{p}$ and $n(1-\hat{p})$ be at least five. In general, you should check this before proceeding. *However, these requirements will be satisfied in all problems encountered in this class.*

Step 2. Enter the data in `Formulas.XLSX` and read the results.

End of Solution Template

Given a desired confidence level, it is possible to control the error term in confidence intervals for proportions (but not in confidence intervals for means) by controlling the sample size. In general, increasing the sample size will decrease the magnitude of the error term (thereby decreasing the length of the confidence interval). The following table shows the sample sizes required for various confidence levels and error terms.

	94%	95%	96%	99%
$\pm 4\%$	552	600	657	1032
$\pm 3\%$	982	1067	1167	1835
$\pm 2.5\%$	1415	1537	2086	2653
$\pm 2\%$	2209	2401	2627	4128
$\pm 1\%$	8836	9604	10506	16512

The first row shows the confidence level. The first column shows the magnitude of the error term. The interior of the table shows the required sample size. Note that halving the error term requires quadrupling the sample size. Small error terms are often too expensive to justify the cost of the requisite extremely large samples. Opinion polls often always use confidence limits of $\pm 2.5\%$ with 95% confidence, giving a sample size of 1537. The $Newsweek$ poll on the health care proposals use a 98.5% confidence interval, with error of $\pm 4\%$.

The researcher is free to decide on a confidence level. Most frequently these levels will be 90% or higher. If the researcher chooses, for example, an 80% confidence level, then the chances of an error – that the confidence interval does not include the true value – are 20%. For most applications, this is an unacceptably high chance of error. If the researcher chooses a confidence level less than 90%, the researcher needs to justify why.

16.2. Example.

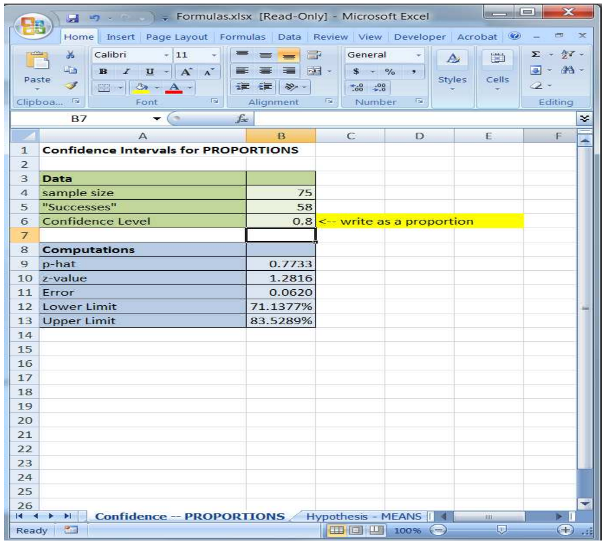
An attorney suspects that the prescription drug Seldane may have harmful side effects. In order to accumulate evidence on this suspicion in support of a possible class action lawsuit, the attorney surveys 75 Seldane users. The survey instrument informs the respondents of the purpose of the survey and then asks if they have experienced harmful side effects. Of the 75 individuals surveyed, 58 respond affirmatively. Find an 80% confidence interval for the proportion saying they suffer harmful side effects. If you were on a jury would you find this evidence convincing? Why or why not?

Solution.

Step 1. First find the sample proportion; since problem is focusing on harmful side effects, "success" is answering "yes" to the question about side effects. Thus

sample size n	75
"successes" k	58
confidence level	80%

Step 2. Now enter the list into the spreadsheet FORMULAS.XLSX, found in the resources section for this course on LEARN.OU.EDU. Note that you will need to select the tab at the bottom labeled `Confidence-PROPORTIONS`. Note also that you should convert the 98% to a proportion, 0.80.

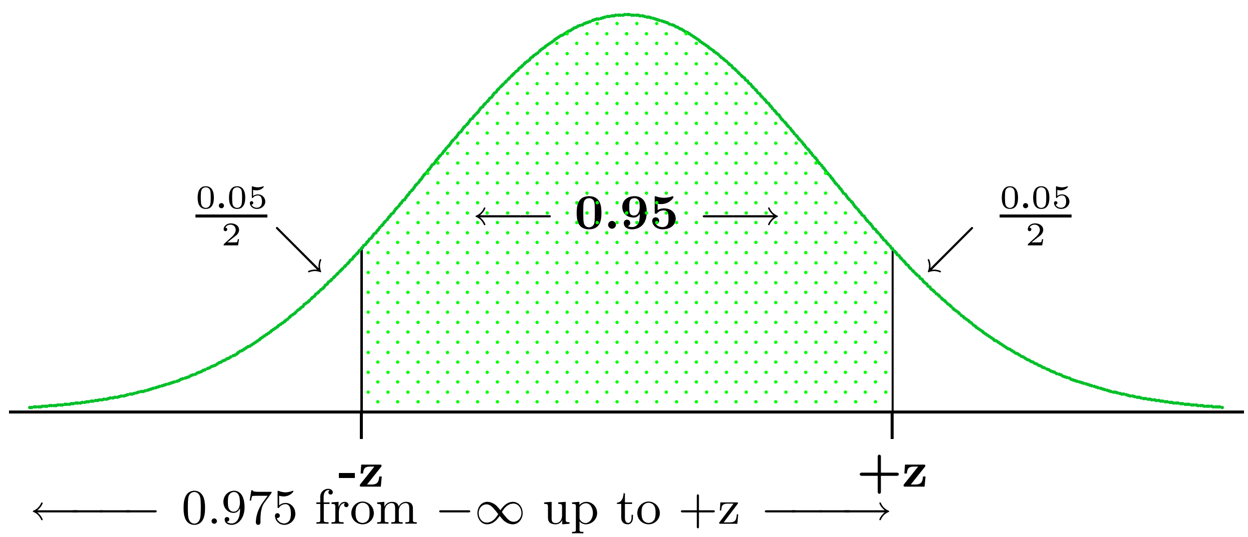


Thus, the confidence interval is:

71.3% to 83.53%

What if you don't have an excel spreadsheet? You can still work the problem, but you will need to use the "inside-out" techniques you learned for normal tables, and the formulae mentioned earlier for the error terms.

Suppose, for example, that you did not have access to Appendix B and needed to find the cut-offs for a 95% confidence interval. You would need to first find z so that 95% of the area under a normal curve falls between $-z$ and $+z$:



If there is 0.95 of the area between $-z$ and $+z$, this leaves 0.05 for the

two tails collectively or 0.025 for each tail individually. Then up to $+z$ there is 0.975 (=0.95+0.025) of the area. We need to compute this last proportion since the normal table is designed to give us areas $up\ to$ a specific z-value.
From the normal table, the z-value which corresponds to 0.975 is $z = 1.96$.
You will always be able to use the Excel spreadsheet on your examination problems.